So far we have discussed probabilities in terms of outcomes either occurring or not occurring, but sometimes the gambler will wish to know the probability of an outcome occurring within a given time or during a given sequence of events.

If a given event must occur exactly twice in 2 years, then if one year is chosen as a unit of time, and the continuous occurrences of the event are called E1, and E2, there are four possibilities:-

Year A	Year B
1.E1,E2	------
2.E1	E2
3.E2	E1
4.-----	E1, E2

Note that these possibilities are similar to the tossing of two coins, and that care has been taken not to reduce the possibilities to three by combining 2 and 3 as one possibility.

It follows that if an event must occur exactly twice only in two years, the probability of it occurring at least once in either year is 3/4. There are three times within the table above in either year that the event occurs only once and one occurrence where it would not occur. Only in possibility 4 does the event not occur in year A. There is a formula to work out these probabilities.

If an event must occur exactly x times in a period divided equally into N smaller periods, then the probability of the event occurring at least once in any small period is: -

=(power(n,x)-(power((n-1),x)))/power(n,x)

Thus, if an event must occur four times in ten years, the probability of it occurring in the first two years is

=(POWER(5,4)-(POWER((5-1),4)))/POWER(5,4)

=((625-256))/625 = (369/625) = 0.5904 or much better than **50%**.

The Chevalier de Mere, a rich Frenchman who liked gambling, was responsible for inviting the philosopher and mathematician Blaise Pascal to carry out some of the earliest work on probability theory.

De Mere played a gambling game in which he bet that he could throw a six in four throws of a die. De Mere progressed from this game to betting that with two dice he could throw a double six in 24 throws. It was known that the odds were in his favour with the first game, and gamblers of the time reckoned that as four is to six (the numbers of ways a die can fall) as 24 is to 36 (the ways two dice can fall), the second game should be favourable. The Chevalier de Mere was not satisfied with this assumption and asked Pascal to work out the true probabilities.

To work out these two problems it is necessary to work out the converse probabilities. The probability of not throwing a 6 in four throws is:-

=power((5/6),4)=625/1296

Therefore the probability of throwing at least one 6 is :-

=1-power((5/6),4) or 0.5177

As an even money proposition, therefore a bet to throw a six in four throws of a die is slightly in favour of the thrower. The probability of not throwing a double six in 24 throws is:-

=power((35/36),24)) = 0.5087

This means that the bet is (slightly) against the thrower.

Various formulae have been put forward to determine the number of throws necessary to make the throwing of a double-6 a better than even chance. The old gamblers' rule in operation in de Mere's time relied on knowing the answer to the lowest number of throws necessary to give a probability of throwing 6 with a single die of more than 1/2. As has been said, this was known to be 4, which might be called the break-even number. According to the rule the break-even number for the double event (N2) is the probability of success on the single event (P1), known to be 1/6 (the probability of a 6 with one die), divided by the probability of success on the double event (P2), known to be 1/36 (the probability of a double-6 with two dice), multiplied by the break-even number of the single event (N1), known to be 4. Thus

n2=(p1/p2)*n1 ***by substitution*** n2=((1/6)/(1/36))x4, or n2=((36/6)x4)

In this equation n2=24, guessed correctly by the Chevalier de Mere to be wrong. Abraham de Moivre, in a book The Doctrine of Chances published in 1716, set out a formula for discovering the approximate break-even point (n) as:-

n=(0.6931/p)

(0.6931 is the "Natural" logarithm of 2). So this gives the answer to our problem as 24.9516. An approximate break even number of 25, which is correct.

This gives an effective break number of 25, which is correct, although de Moivre's formula gives too high a number, which is only slightly too high when the probability is small, but may be critically too high when the probability is larger. For example, de Moivre's formula for the break-even number for throwing a 6 with one die gives n = 0.6931 x 6 = 4.1586, which gives an effective break-even number of 5, which is too high, 4 being correct, as we have found.

It is interesting that John Scarne, in his famous book Scarne's Complete Guide to Gambling, published in 1961, states that the odds to one should be multiplied by 0-6931. Thus his approximate answer to de Mere's problem is n = 0-6931 x 35 = 24.2585. This answer is too low, although it gives the correct effective break-even number as 25. Scarne's formula applied to throwing a 6 with a single die gives the answer n = 0-6931 x 5 = 3.4655, again slightly low, but giving the correct break-even number of 4. The third formula to find the break-even number is as follows:-

If the probability of the outcome is (1/a) then :-

$$n=(\log 2/((\log a)-(\log(a-1))))$$

In this case, the logarithms are the ordinary logarithms, to base 10.
The break-even number for throwing a double 6 thus becomes:-

$$n = (\log 2/(\log 36 - \log 35))$$

which, with the use of four figure log tables, gives n a value of 24.6721, thus giving 25 as the effective break-even number. For throwing a 6 with one die, n = 3.8005, confirming 4 as the effective number.

When the Chevalier de Mere asked Pascal to help with this problem Pascal provided him with his answer, and began a correspondence about probabilities with Pierre de Fermat, another French mathematician, which established probability theory as a new branch of mathematics.