Converse - "Something that has been reversed; an opposite."

Often when you work out the probability of an event, you sometimes do not need to work out the probability of an event occurring, in fact you need the opposite, the probability that the event will not occur. For example, The probability of throwing a 1 on a die is 1/6 therefore the probability of a 'non 1' is (1-1/6) which equals 5/6. Converse probabilities are used to work out such problems such as*:* -

"What is the probability of exactly one soccer match ending in a draw within a group of three separate matches?"

Let us assume the chance of a draw occurring in any match is 1/3 or 33.33%. To fulfil our target of only one match ending in a draw we would require the other matches not to end in a draw or (1-(1/3)) which equals 2/3 or 66.66%. Therefore the probability of the first match out of three being a drawn and the other two not being a draw is 1/3x2/3x2/3 which equals = 4/27 or (.33*.67*.67) = 14.81%.

In our group of three matches there are three ways for only one match to draw, DXX, XDX, XXD, therefore we need to add together all the probabilities, three in this case.

The final answer to the probability of one match drawing is (4/27)+(4/27)+(4/27) = 4/9 or (=.1481+.1481+.1481) = 44.44%.

When I first started filling out the football pools coupon for my father a long time ago I struggled with this calculation. Many thanks to Michael Szecepaniak from Colorado for reminding me how I first checked my calculations. Michael suggested I post the table he used to round off the above section. Here you can see clearly how the calculation relates to each underlying event. Twelve matches tied = 12/27 = 44.44%

	WWW
LWW	
TWW	

WWL
LWL
TWL

WWT
LWT
TWT

WLW
LLW
TLW

WLL
LLL
TLL

WLT
LLT
TLT

WTW
LTW
TTW

WTL
TLL
TTL

WTT
LTT
TTT

The Birthday Problem

Converse probabilities are used to work out the infamous birthday problem. Many people find the answer puzzling but it can be proved by either asking your personal manger for birthday dates or flicking through a the who’s who in your reference library. I've used this piece of probability on a number of occasions to completely fox a room of participants. It's good entertainment for a coach or plane trip.

The question is:-

"How many people should be gathered in a room together before it is more likely than not that two of them share the same birthday?"

Ignoring the issues of leap years the problem is solved as follows:-

When the first person enters the room and announces their birthday, the probability of the second person sharing the same birthday is 1/365. Conversely, the probability of the second birthday being different is the opposite of the first calculation, 364/365. When two birthdays are known, the probability of the third being different is 363/365, as there are now two 'favourable' outcomes among 365. The compound probability of birthday 2 being different from birthday 1, and of birthday 3 being different from the other two, these being independent outcomes, is:-

(364/365)*(363/365) = 0.991796 or 99.2% chance that two people will not share the same birthday.

Note the start of the sequence is (365/365). We have removed this as it does not affect the result of the calculation.

All that is necessary now is to continue adding terms to the fraction until it equals less than 1/2 or 50%, since as soon as the probability is less than 1/2 that all birthdays are different, the probability is clearly more than 1/2 that any two are the same. In other words it is more likely than not that two people in the room share the same birthday. The following chart shows the number of the people in the room and the probability that they**DO NOT** share the same birthday.

People

Chance %

2

99.7

3

99.2

4

98.4

5

97.3

6

96.0

7

94.4

8

92.6

9

90.5

10

88.3

11

85.9

12

83.3

13

80.6

14

77.7

15

74.7

16

71.6

17

68.5

18

65.3

19

62.1

20

58.9

21

55.6

22

52.4

23

49.3

24

46.2

50

3.0

100

3,254,690 to 1 on

The fraction drops to less than 1/2 with 23 iterations, so it is more likely than not that in any gathering of 23 or more persons, two of them will share a birthday. Only 50 people need be present for the 'coincidence' of two of them having the same birthday to become, roughly, a 30-1 on chance. In a company of 100 employees the odds are more than three million to one on that two share a birthday.

The birthdays proposition is one where a gambler who can estimate probabilities can make money from an unsuspecting audiance.