

One of the important steps you need to make when considering the probability of two or more events occurring. Is to decide whether they are independent or related events.

Examples:-

Mutually Exclusive vs. Independent

It is not uncommon for people to confuse the concepts of mutually exclusive events and independent events.

Definition of a mutually exclusive event

If event A happens, then event B cannot, or vice-versa. The two events "it rained on Tuesday" and "it did not rain on Tuesday" are mutually exclusive events. When calculating the probabilities for exclusive events you add the probabilities.

Independent events

The outcome of event A, has no effect on the outcome of event B. Such as "It rained on Tuesday" and "My chair broke at work". When calculating the probabilities for independent events you multiply the probabilities. You are effectively saying what is the chance of both events happening bearing in mind that the two were unrelated.

To be or not to be.....?

So, if A and B are mutually exclusive, they cannot be independent. If A and B are independent, they cannot be mutually exclusive. Simple isn't it? Or is it? This is where a lot of people go wrong in trying to work out probabilities as sometimes the status of two sets of probabilities are not as clear cut as it seems.

If the events we chose were "it rained today" and "I left my umbrella at home"; they are not necessarily mutually exclusive, but they are probably not independent either, because one would think that you'd be less likely to leave your umbrella at home on days when it rains. Also, think about the stock market. Prices on individual stocks should reflect the underlying characteristics of an investment and its return. However it's quite common for stocks to rise and fall in a manner that doesn't appear related to the true underlying value. The debate on how perfect the stock market is, in terms of pricing efficiency, continues to be debated.

Those facts aside use the following to help your understanding of the definition: -

Example of a mutually exclusive event

What happens if we want to throw 1 and 6 in any order?

This now means that we do not mind if the first die is either 1 or 6, as we are still in with a chance. But with the first die, if 1 falls uppermost, clearly it rules out the possibility of 6 being uppermost, so the two Outcomes, 1 and 6, are exclusive. One result directly affects the other. In this case, the probability of throwing 1 or 6 with the first die is the sum of the two probabilities, $1/6 + 1/6 = 1/3$.

The probability of the second die being favourable is still $1/6$ as the second die can only be one specific number, a 6 if the first die is 1, and vice versa.

Therefore the probability of throwing 1 and 6 in any order with two dice is $1/3 \times 1/6 = 1/18$. Note that we multiplied the the last two probabilities as they were independent of each other!!!

Example of an independent event

The probability of throwing a double three with two dice is the result of throwing three with the first die and three with the second die. The total possibilities are, one from six outcomes for the first event and one from six outcomes for the second, Therefore $(1/6) * (1/6) = 1/36$ th or 2.77%.

Independent or mutually exclusive event?

The two events are independent, since whatever happens to the first die cannot affect the throw of the second, the probabilities are therefore multiplied, and remain $1/36$ th.