

One of the most valuable uses to which a gambler can put his knowledge of probabilities is to decide whether a game or proposition is fair, or equitable. To do this a gambler must calculate his 'expectation'.

A gambler's expectation is the amount he stands to win multiplied by the probability of his winning it. A game is a fair game if the gambler's expectation equals his stake. If a gambler is offered 10 units each time he tosses a head with a true coin, and pays 5 units for each toss is this a fair game? The gambler's expectation is 10 units multiplied by the probability of throwing a head, which is $1/2$. His expectation is 10 units \times $1/2$ = 5 units, which is what he stakes on the game, so the game is fair.

Suppose a gambler stakes 2 units on the throw of a die. On throws of 1, 2, 3 and 6 he is paid the number of units shown, on the die. If he throws 4 or 5 he loses. Is this fair? This can be calculated as above. The probability of throwing any number is $1/6$, so his expectation is $6/6$ on number 6, $3/6$ on number 3, $2/6$ on number 2 and $1/6$ on number 1.

His total expectation is therefore $6/6+3/6+2/6+1/6$, which equals 2, the stake for a throw, so the game is fair.

Prospects of ruination

When we talk of fair games, there is another aspect to the question which experienced gamblers know about, and which all gamblers must remember. If a game is fair by the above criterion, it nevertheless is still true that if it is played until one player loses all his money, then the player who started with most money has the better chance of winning. The richer man's advantage can be calculated.

The mathematics required to arrive at the full formula are complex, but it can be shown that if one player's capital is c and another player's is C , then the probability that the player who began with c is ruined in a fair game played to a conclusion is $C/(c+C)$ and therefore that the probability that he will ruin his opponent is $c/(C+c)$.

Suppose player X, with 10 units, plays another player, Y, with 1,000 units. A coin is tossed and for each head player X pays player Y one unit, and for each tail player Y pays player X one unit. The probability of player X ruining player Y is $10/(1000+10)$ or $1/101$. Player Y has a probability of $100/101$ of ruining his opponent, An advantage of over 99%.

This overwhelming advantage to the player with the larger capital is based on a fair game. When a player tries to break the bank at a casino, he is fighting the house edge as well as an opponent with much larger resources. Even a small percentage in favour of the casino reduces the probability of the player ruining the casino to such an extent that his chance of profiting is infinitesimal. If you are anticipating playing a zero sum game then make sure you are aware of the prospect of ruin.